A Proof Design for Proofs of "$A \subseteq B$"

Let A and B be sets.

The definition of "$A \subseteq B$" has two equivalent forms:

__Def'n 1:__

$A \subseteq B \Leftrightarrow \forall x \in U$, if $x \in A$, then $x \in B$.

__Def'n 2:__

$A \subseteq B \Leftrightarrow \forall x \in A,\ x \in B$.

These are equivalent because the actual domain of x in Def'n 2 is the same set A as the effective domain of x in Def'n 1.

__To Prove:__ $A \subseteq B$

__Proof:__ Let $x \in A$ be given.

[N.T.S.: $x \in B$]

$\vdots$

$\therefore x \in B$.

$\therefore A \subseteq B$, by Direct Proof.

QED

Proof Design

EXAMPLE PROOF

__EXAMPLE:__

__TO PROVE:__ For all sets A and B,

$(A \cap B) \subseteq A$.

__Proof.__

Let A and B be any sets.

[N.T.S.: $(A \cap B) \subseteq A$.]

Let $x \in A \cap B$ be given.

$\therefore x \in A$ AND $x \in B$, by def'n of "intersection".

$\therefore x \in A$, by Specialization.

[$\therefore \forall x \in A \cap B,\ x \in A$].

$\therefore (A \cap B) \subseteq A$, by Direct Proof.

$\therefore$ For all sets A and B, $(A \cap B) \subseteq A$, by Direct Proof.

QED.